

The Four Worlds Paradox

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June 18, 2010

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This essay examines Nathan Salmon's Four Worlds Paradox and critiques two solutions, one proposed by Salmon himself and the other by Graeme Forbes. It argues that Forbes's is the more plausible solution while urging a different approach to the problem of vagueness that gives rise to the paradox in the first place.

The Four Worlds Paradox is Salmon's name for his version of a puzzle, first raised by Chandler, that seems to reveal an inconsistency in S5 modal theory. We assume that for any object made from some hunk of matter, there exists a possible world in which the same object is made from a hunk of matter that partially overlaps the original hunk but no possible world in which the object is made from a non-overlapping hunk. Suppose the object is a ship and the hunk of matter is an aggregate of 100 planks. We stipulate that if two planks are replaced, the same ship continues to exist, but if three or more planks are replaced, the result is a numerically distinct ship. Call the original ship a and a ship made by substituting some of the planks b . If b is made from all except two of the same planks, then $a = b$, but if three planks are different, then $a \neq b$. It can then be demonstrated, using an argument valid in S5, that there exist at least two possible worlds w_i and $w_{(i+1)}$ such that a ship a in w_i and a ship b in $w_{(i+1)}$ are made from exactly the same planks but $a \neq b$.¹

¹ Nathan Salmon, *Reference and Essence* (Princeton, NJ: Princeton University Press, 1981), 230-31.

The argument generates the same conclusion no matter how the threshold of identity is adjusted. All that we must assume is that a threshold must exist—that, loosely speaking, two objects having no parts in common are distinct and two objects having all parts in common are identical.² Such an assumption seems intuitively unquestionable. Since the argument, in reliance on that assumption, proves the possible existence of two non-identical ships made of exactly the same parts, we have a paradox—an apparently valid argument from apparently true premises to an apparently absurd conclusion.

Salmon's solution is to revise the axioms of S5, eliminating the assumption that accessibility between possible worlds is transitive. Given a set of possible worlds w_1, w_2, \dots, w_n , it is ordinarily assumed that if for each $w_i, w_{(i+1)}$ is accessible to w_i , then w_n is accessible to w_1 . Salmon suggests that this may not be the case, that at some point in the sequence there is a w_i that is not accessible to w_1 , even though it is accessible to $w_{(i-1)}$ and $w_{(i-1)}$ is itself accessible to w_1 . In modal symbolism, this means we cannot infer $\diamond A$ from $\diamond\diamond A$, or in other words, what is possibly possible relative to a given world may be impossible relative to that world. To say that it is possibly possible is just to say that it could become possible, not that it actually is possible. Without the transitivity of accessibility, the Four Worlds argument becomes invalid and this resolves the paradox.³

Forbes's solution is to endorse counterpart theory.⁴ This holds that if two objects a and b inhabiting different worlds differ at all in their composition, then $a \neq b$, i.e. they are not the same object but only, at most, similar objects. If there is some sense in which a could have been b , then we say that b is a counterpart of a , but the counterpart

² Additional qualifications, such as time, place, or agent of construction, do not affect the argument, since these can all be included in the replacement process.

³ Salmon, 238-40.

⁴ Graeme Forbes, "Two Solutions to Chisholm's Paradox," *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition* 46, no. 2 (1984). <http://www.jstor.org/stable/4319699> (accessed June 5, 2010).

relationship is not identity. Thus in the Four Worlds argument, every plank substitution results in a different ship and so each premise to the contrary is false. This renders the argument unsound and so resolves the paradox.

In assessing their relative plausibility, we begin by noting that Salmon and Forbes both appeal to analogous sorites paradoxes for support of their positions, an appeal that nicely illuminates the relevant problems in possible-worlds interpretations of modal logic. We set up a typical sorites paradox thus. We note that a person with no hair on his head is by definition bald, and so is a person with only one hair on his head. We generalize that the addition of one hair does not suffice to make a bald person non-bald. Let B_{an} be the proposition that a person a with n hairs on his head is bald. Then B_{a0} holds, B_{a1} holds, and in general, if B_{an} holds, then so does $B_{a(n+1)}$. In other words, $B_{an} \rightarrow B_{a(n+1)}$. Continuing in this manner, in due course we infer that $B_{a(100,000)}$ holds. However, the average human, who is not bald, has about 100,000 hairs on his head. Salmon's solution invokes multi-valued logic with degrees of truth, which changes the truth value of material implication, so that given B_{an} and $B_{an} \rightarrow B_{a(n+1)}$, $B_{a(n+1)}$ does not necessarily follow, contrary to our intuition about the validity of modus ponens. By similar reasoning, Salmon shows that we can dispense with the transitivity of accessibility, since accessibility too admits of degrees of truth and so can be analyzed with the methods of multi-valued logic.

Forbes also employs multi-valued logic, but applies it to the modal operator rather than the accessibility relation. His solution, he says, "works by introducing degrees of *de re* possibility, *not* degrees of identity, in order to render 'possibly being identical to α ' a complex predicate of degree."⁵ Modus ponens fails in the sorites paradox

⁵ Forbes, 175.

because a person with $n+1$ hairs is not quite so bald as a person with n hairs, and at some point “not quite so bald” becomes “not bald at all.” Similarly, given worlds w , w_n and $w_{(n+1)}$, something may be less possible, relative to w , in $w_{(n+1)}$ than it is possible relative to w_n , and over enough transitions the relative possibility may diminish to impossibility.

To the question of which solution is more plausible, within the context of possible-worlds semantics, I think Graeme has the edge in addressing the vagueness issue insofar as it carries over from the sorites to the Four Worlds paradox. Salmon appears to be treating identity as vague, so that there may be instances where the identity relationship is indeterminate. As Graeme points out, this requires an arbitrary choice of some n where $B_{an} \rightarrow B_{a(n+1)}$ fails to hold, while his solution treats all the conditionals alike.⁶ To the objection that counterpart theory rigidly essentialist, Graeme responds that this depends “only on whether or not it is consistent with the truth of some instance of the *object* language schema ‘ x exists and is F and possibly exists and is not- F ’.”⁷ So long as it is consistent in this sense, counterpart theory is not committed to denying that objects have contingent properties.⁸

At the same time, I think both are missing a vital point about vagueness, and the point is equally germane to questions of the “Is b identical with a ?” kind and the “Does a have property F ?” kind. The point is that the right answer may depend entirely on the answer to another question: “Why do we want to know?”

A person with 5,000 hairs on his head has those 5,000 hairs, and that is a fact no matter whether we call him bald or hirsute. The label we stick on him won’t change that

⁶ Forbes, 176-7.

⁷ Forbes, 179.

⁸ Ibid.

fact. It could conceivably change the way we treat him in certain ways. If we decide he is bald, then maybe we'll call him names like "Baldy" or "Chrome Dome." Maybe there are other ways we'll treat him differently than we treat hirsute men, or maybe not; but the ultimate arbiter in defining any word is usage, not philosophy, and usage is just the collective decisions of all the people who use the language to which the word belongs. Our language was not given to us along with a commission to seek out, by the exercise of our ingenuity, the true definition of every word. We invented it ourselves at some point in the process by which we evolved into human beings. Being its inventors, it is entirely up to us how we use it. There are no rules except the rules we make ourselves.

If we ourselves are not sure whether a particular person is bald, one possibility we need to consider is that the question just might not be appropriate—that in some situations, if we have to ask, then there is no uniquely correct answer, no fact of the matter that will arbitrate between right answers and wrong answers. We may wish it were otherwise, but wishing will not make it so. We would like a uniquely correct answer to every philosophical question, but the universe is not obliged to give us everything we'd like. For some propositions, such as "Fred is bald," our own judgment is the only truthmaker. Is Fred bald or not? Why do we need to decide? What will be the consequences of our decision? Is there something we ourselves must do differently if he is bald than if he is not bald? Analogous questions apply to Salmon's ship. Is it the same ship or a different ship if we replace three planks? Why are we asking? Is there something we ourselves must do differently in the one case than in the other case?

Assuming that we come up with some answer to the "Why are we asking?" question, then our decision as to whether Fred is bald or whether we have produced a different ship is bound to be arbitrary, and as philosophers we have good reasons for

abhorring arbitrary answers. But their occasional necessity is established by precedent in the legal world—which, no less than the community of philosophers, tries very hard to avoid arbitrariness. Maturity is a famously vague concept, but vitally relevant to many issues of governance. The law cannot treat 10-year-olds the same as it treats 30-year-olds because a 10-year-old is not mature and a 30-year-old presumptively is. So, what about 20-year-olds? It is common knowledge that some 20-year-olds are more mature than the average 30-year-old while some are barely more mature than the average 10-year-old. So what is the law to do? In the current United States, the law has decided that in general, an 18-year-old is mature enough to be treated as an adult, one exception being with regard to consumption of alcoholic beverages.

Of course the law is not philosophy, and legislators must work under constraints that do not bind philosophers. But the relevant commonality is that when someone asks, “What difference does it make whether Johnny is mature?” we have an answer, and that answer not only compels us to make a decision about when to declare Johnny an adult but also informs that decision, even if it does not eliminate its arbitrariness. In this case, the answer to a question depended on the reason for asking it, and philosophers should consider the possibility that the same could be true of some of their favorite questions.

References

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(The original version of this essay was written for a class in the Philosophy of Logic and Mathematics taught by Dr. Matthew Davidson at California State University, San Bernardino.)

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(This page was last updated on November 25, 2010.)